

A low α_s : hint of new physics at the GUT scale?

Mar Bastero-Gil^a and Biswajoy Brahmachari^b

(a) Scuola Internazionale Superiore di Studi Avanzati

34013 Trieste, ITALY.

(b) International Centre For Theoretical Physics,

34100 Trieste, ITALY.

Abstract

In a SUSY GUT having an extra reverse doublet-triplet splitting near the GUT scale, where the mass of an extra doublet is greater than the mass of an extra triplet by two orders of magnitude, a low prediction of α_s can be achieved through threshold corrections via the heavy scalars in the fundamental representations, making the prediction consistent with the values being suggested by low energy measurements. We implement this mechanism in a SUSY SU(5) GUT minimally extended to suppress Higgsino mediated proton decay. We also point out that as a consequence of this extension a natural scenario arises with a large hierarchy in Yukawa couplings ($\lambda_t/\lambda_b \sim 40$). An experimental low value of $\alpha_s(m_Z)$ along with the non-observation of dimension-five proton decay modes at SuperKamiokande detector will favor such an extension of the SUSY SU(5) GUT over the minimal case.

I. INTRODUCTION

The value of α_s , calculated from a global fit of the LEP/SLC data assuming Standard Model (SM) is correct, is three standard deviations away from the value calculated from the low energy experiments. Some time ago it was believed that the discrepancy is due to the error in the low energy measurements. However, with the increasing rigor of the lattice QCD calculations along with the wealth of low energy data, a real difference between the high and low energy predictions is emerging [1]. Even though the low energy experiments are indicating a value of $\alpha_s = 0.112$, the global fits at the Z-peak from the LEP/SLC data assuming only the SM particle content and interactions suggests a value of $\alpha_s \simeq 0.125$. Moreover, a high value of α_s like 0.125 leads to a high value of $\Lambda_{QCD} \simeq 500$ MeV, differing from the measurement of the same from the perturbative QCD and sum rules which is like 200 MeV. Indeed the work of Kane, Stuart and Wells [2] propose that a combined fit of LEP/SLC data including the SUSY particles and interactions can lower the value of α_s . They have shown that to fit $\alpha_s = 0.112$ the χ^2 minimization requires a chargino mass near 80 GeV and a stop mass near 60 GeV. We note that, in stark contrast, to predict a low value of $\alpha_s \sim 0.11$ in a minimal SUSY SU(5) GUT, the mass scale of the superpartners has to be considerably higher than the electroweak scale.

It is well known by now, that the coupling constant unification in SUSY SU(5) model predicts a high value of $\alpha_s \sim 0.126$ (0.130) for a choice of the SUSY breaking scale $M_{SUSY} = 1$ TeV (500 GeV). Light threshold effects tend to increase the prediction of α_s even higher than the value obtained from LEP data in the step function approximation or the so called run and match method [3]. When an improved treatment of the low energy threshold W effects is done, including not only the leading logarithmic contributions but also the finite part of the diagrams, the prediction of α_s increases even further [4,5]. At the same time when the heavy spectrum is non-degenerate, the heavy threshold correction to the prediction of α_s comes from the split incomplete SU(5) multiplet containing the color triplet Higgs field [6]. This correction has an increasing effect on α_s whenever the mass of the color triplet

is greater than the mass of the doublet. It has been noted [7–9] that there exist stringent lower bounds on the mass of the color triplet coming from Higgsino mediated proton decay and hence in the minimal model the doublet-triplet splitting sizably increases the predicted value of α_s again.

There has been a number of attempts to lower the prediction of α_s in a supersymmetric GUT. In the low energy scales the Winos and the Binos give a threshold correction to α_s which has an opposite sign to the threshold correction induced by gluinos; consequently, it has been pointed out by Shifman and Roszkowski [10] that if one gives up the unification of the gaugino masses a low prediction of α_s can be obtained due to a large correction coming from the mass difference between the charginos and the gluinos. On the other hand by invoking an intermediate B-L symmetry breaking scale can also get a low prediction of α_s . Such a scenario has been explored in Ref [11] by introducing an enlarged scalar sector inspired by superstring theory. Another possibility is to introduce higher dimensional SU(5) Higgs multiplets. If the 50, $\overline{50}$ and 75 dimensional Higgs fields of SU(5) are introduced [12], the heavy threshold effects coming from the mass splitting within these extra multiplets can also lower the prediction of α_s [9].

In this paper we stick to the conventional one step breaking of a SUSY GUT model without giving up the universality of the gaugino masses [13] at the unification scale and consider the possibility of achieving a low α_s . We explore a possible reverse doublet-triplet splitting which will have an effect opposite to the conventional doublet-triplet splitting on the prediction of α_s . Such a strange reverse doublet-triplet splitting is indeed possible in a realistic SU(5) model as will be displayed in this letter.

This paper is organized as follows. In section II we give the mechanism, in section III we implement it in a model, in section IV we discuss m_t/m_b in this new model, in section V we note some observations regarding the model and in section VI we conclude.

II. MECHANISM

At first let us consider the prediction of α_s including the threshold effects in SUSY SU(5), which is well-studied in the literature [4,5,8,9,14]. Throughout this paper we will assume that including the threshold corrections, the minimal SUSY SU(5) GUT predicts $\alpha_s = 0.126$ [5]; we will also assume that the mass of the color triplet Higgs scalars in a minimal SU(5) GUT is $10^{16.6}$ GeV*. In particular the prediction of α_s in the minimal SUSY SU(5) can be written as,

$$\alpha_s^{-1}(m_Z) = \frac{1}{2} [3\alpha_2^{-1}(m_Z) - \alpha_1^{-1}(m_Z)] - \frac{3}{5\pi} \ln\left[\frac{M_3}{M_2}\right] + T_L + \delta_{2loop}, \quad (1)$$

where, M_3 and M_2 are the masses of the triplet and the doublet Higgs scalars present in the 5 and $\bar{5}$ representations of SU(5), and T_L parametrizes the contribution from all other light degrees of freedom (excluding the light Higgs doublets), and in a simple step function approximation [8] $T_L = \frac{1}{2\pi} \ln \frac{M_{SUSY}}{m_Z}$. M_{SUSY} can be considered in the simplest approach as a common SUSY breaking scale, or as an effective SUSY mass parameter [3] resumming the effect of the detailed SUSY spectrum, and in this sense it can be either more or less than m_Z depending on the super-partner masses. Here we assume that the supersymmetric particle masses are not much higher than 1 TeV and hence, the term T_L is not enough to lower the prediction of α_s consistent with the low energy experiments. The term δ_{2loop} parametrizes the two loop corrections (due to the light as well as the heavy degrees of freedom). In a generic situation one has $M_3 > M_2$ and consequently the doublet-triplet splitting increases the prediction of α_s via the second term of Eqn (1). However, we notice the hypothetical possibility, that if the mass of the doublet were more than the mass of the triplet, we would have had a reverse effect on α_s . Keeping this in mind we add one more 5 + $\bar{5}$ Higgs scalars

* This stringent lower bound comes from the non-observation of the dimension five proton decay processes assuming $M_{SUSY} \leq O(TeV)$ [4,8]. We are taking the lowest allowed value of M_3 which leads to the lowest $\alpha_s(m_Z)$.

with doublet and triplet masses as M'_2 and M'_3 GeV respectively. In that case the Eqn (1) gets modified to,

$$\alpha'^{-1}_s(M_Z) = \frac{1}{2} [3\alpha_2^{-1}(m_Z) - \alpha_1^{-1}(m_Z)] - \frac{3}{5\pi} \ln\left[\frac{M_3 M'_3}{M_2 M'_2}\right] + T_L + \delta'_{2loop}. \quad (2)$$

Taking the difference of Eqn (1) and Eqn (2) and assuming,

$$M_3 = 10^{16.6} ; M_2 = 10^2 ; M'_3 = 10^x ; M'_2 = 10^y, \quad (3)$$

we get,

$$\Delta\alpha_s^{-1} \equiv \alpha'^{-1}_s(m_Z) - \alpha_s^{-1}(m_Z) = \frac{3}{5\pi} (y - x) \ln 10 + [\delta_{2loop} - \delta'_{2loop}]. \quad (4)$$

It is easy to check from Eqn (4) that taking $y - x = 2.26$ we can get $\Delta\alpha_s^{-1} = 0.99$ and consequently α_s decreases by 11%, from 0.126 to 0.112. We have neglected the *difference* in the 2 loop terms which is due to the difference between the two heavy sectors only. Instead if we add n extra pairs of $5 + \bar{5}$ the required splitting in each SU(5) multiplet is only 2.26/n orders of magnitude.

It is important to check the change in the gauge boson masses due to this extra splitting, because, as a general trend [see the formula below] a reduction in the predicted value of α_s is associated with a reduction in M_V which is the mass of the heavy gauge bosons mediating the dimension six proton decay processes. In the minimal SUSY SU(5) GUT the the mass of the heavy gauge bosons can be predicted from the following formula,

$$\ln \frac{M_V}{m_Z} = \frac{\pi}{2} [\alpha_1^{-1}(m_Z) - \alpha_2^{-1}(m_Z)] + \frac{1}{10} \ln\left[\frac{M_3}{M_2}\right] - \frac{1}{2} \ln\left[\frac{M_\Sigma}{m_Z}\right] + T_L^1 + \Delta_{2loop}, \quad (5)$$

where, T_L^1 parametrizes the threshold effects coming from the fields present in the low energy scales, and M_Σ is the mass of the heavy Higgs scalar in the adjoint representation. In a simple step function approximation, $T_L^1 = -\frac{5}{12} \ln \frac{M_{SUSY}}{m_Z}$. At this stage we introduce the extra $5 + \bar{5}$ multiplets. Now Eqn (5) looks as,

$$\ln \frac{M'_V}{m_Z} = \frac{\pi}{2} [\alpha_1^{-1}(m_Z) - \alpha_2^{-1}(m_Z)] + \frac{1}{10} \ln\left[\frac{M_3 M'_3}{M_2 M'_2}\right] - \frac{1}{2} \ln\left[\frac{M_\Sigma}{m_Z}\right] + T_L^1 + \Delta'_{2loop}. \quad (6)$$

We compare with minimal SUSY SU(5) case as before and obtain,

$$\ln\left[\frac{M'_V}{M_V}\right] = -\frac{1}{10}(y-x)\ln 10 + [\Delta_{2loop} - \Delta'_{2loop}]. \quad (7)$$

Using $y - x = 2.26$ and neglecting $\Delta_{2loop} - \Delta'_{2loop}$, we have,

$$\frac{M'_V}{M_V} = 10^{-0.226}. \quad (8)$$

Clearly, this is a small reduction, and is consistent with the bounds on M'_V from dimension six proton decay. In practice, Eqn (5) and Eqn (6) only determines the combination $\frac{M_V'^2 M_\Sigma}{m_Z^3}$. The change in M_V can in principle be compensated by a suitable lowering of M_Σ too. On the other hand, M_Σ is determined in terms of M'_V modulo the unknown Yukawa coupling of the 24^3 term in the superpotential. In a natural scenario M_Σ is not expected to be much lower than M'_V . In the present case, the smallness of the change in M_V due to the introduction of the extra $5 + \bar{5}$ scalars assures that the fine-tuning of the mass M_Σ is not necessary.

If the extra triplet couples to the fermions it leads to dimension-five proton decay diagrams and consequently its mass is bounded from below to $10^{16.6}$ GeV. In such a situation the extra doublet has to have a mass at the Plank scale to lower the prediction of α_s . This rigid situation can be relaxed if we introduce a mechanism to suppress the Higgsino mediated proton decay in a SU(5) theory so that the mass of the triplet can safely be lowered below the $10^{16.6}$ GeV scale. We will discover below, that the mechanism to suppress Higgsino mediated proton decay naturally introduces extra $5 + \bar{5}$ Higgs scalars also.

III. MODEL

Babu and Barr [15] have shown that it is possible to suppress the Higgsino mediated proton decay strongly in an SO(10) model by a judicious choice of the fields, couplings and VEVs at the GUT scale. Here, we consider a similar scenario in a SUSY SU(5) GUT. Consider an SU(5) invariant superpotential involving the three pairs of scalar superfields $\bar{5}_i$ and 5_i in the fundamental representation, the Σ superfield in the adjoint representation,

and three singlet superfields η_1 , η_2 and η_3 . The fermions are in the usual $\overline{5}_F$ and 10_F representations of the SU(5) group. The superpotential is,

$$\begin{aligned}
W = & \overline{5}_1(M_{12} + \lambda_{12} \Sigma)5_2 + \overline{5}_2(M_{21} + \lambda_{21} \Sigma)5_1 \\
& + \overline{5}_2\lambda_{23} \eta_1 5_3 + \overline{5}_3\lambda_{32} \eta_1 5_2 + \overline{5}_3\lambda_{33} \eta_2 5_3 \\
& + M \Sigma^2 + \beta \Sigma^3 \\
& + \lambda_1^\eta \eta_2^2 \eta_3 + M_{13}^\eta \eta_1 \eta_3 \\
& + \lambda_d \overline{5}_1 \overline{5}_F 10_F + \lambda_u 5_1 10_F 10_F.
\end{aligned} \tag{9}$$

The Z_N symmetry which allows only these couplings and forbids the rest of the SU(5) invariant couplings in the scalar sector is given in Table I.

We notice that only 5_1 and $\overline{5}_1$ fields can couple to the fermions and they do not have a direct mass term. This suppresses the Higgsino mediated proton decay. The scalars Σ , η_1 , η_2 get VEVs at the GUT scale whereas η_3 does not get a VEV. The singlet η_3 couples to η_1 and η_2 only and it is required to guarantee a consistent set of minimization conditions. The form of the mass matrix can be easily calculated from the superpotential in Eqn (9). The general form of the mass matrices of the triplet (M^T) and the doublet (M^D) is given by,

$$M^T = \begin{pmatrix} 0 & a_3 & 0 \\ b_3 & 0 & c \\ 0 & d & e \end{pmatrix}, \quad M^D = \begin{pmatrix} 0 & a_2 & 0 \\ b_2 & 0 & c \\ 0 & d & e \end{pmatrix} \tag{10}$$

To generate one vanishing eigenvalue we need to keep the usual form of the fine-tuning of the minimal SU(5) model in the doublet sector to the relation,

$$b_2 = M_{21} - 3\lambda_{21}v = 0, \tag{11}$$

where we have used,

$$\langle \Sigma \rangle = \text{diag} (2v, 2v, 2v, -3v, -3v). \tag{12}$$

All these eigenvalues receive corrections of the order of $m_{3/2}$ due to the soft breaking of supersymmetry. Now the model has three heavy triplets, two doublets having masses d_1 and

d_2 at the unification scale and one doublet of mass of order $m_{3/2}$. Following the definition in Eqn (4), and denoting the triplet-mass[†] in the minimal SU(5) case as M_3^0 ,

$$\Delta\alpha_s^{-1} = \frac{3}{5\pi} \ln \left| \frac{M_3^0 d_1 d_2}{\text{Det } (M^T)} \right| = \frac{3}{5\pi} \ln \left| \frac{M_3^0 \sqrt{a_2^2 c^2 + c^2 d^2 + e^2 a_2^2}}{a_3 b_3 e} \right|. \quad (13)$$

IV. TOP AND BOTTOM QUARK MASS SPLITTING

At this point we note that in this model there is an interesting possibility to generate a hierarchy of Yukawa couplings to the fermions in the up and in the down sector. When the SU(5) symmetry is broken there is one combination of the 5_i fields and one combination of $\bar{5}_i$ fields remaining light. Denoting them with a superscript zero we can explicitly write down the combinations as,

$$\begin{aligned} 5_1^0 &= 5_1, \\ \bar{5}_1^0 &= \gamma_1 \bar{5}_1 + \gamma_2 \bar{5}_2 + \gamma_3 \bar{5}_3. \end{aligned} \quad (14)$$

It is straightforward to calculate the coefficients γ_i from the doublet mass matrix. In particular,

$$\gamma_1 = 1 / \sqrt{1 + \frac{a_2^2}{d^2} + \frac{a_2^2 e^2}{d^2 c^2}}. \quad (15)$$

Noting that at the GUT scale only the 5_1 and $\bar{5}_1$ couple to the fermions, we can write down the effective Yukawa couplings of fermions to the $\bar{5}_1^0$ and 5_1^0 scalars, as,

$$\lambda_u^{eff} = \lambda_u \quad ; \quad \lambda_d^{eff} = \gamma_1 \lambda_d. \quad (16)$$

The doublets residing in the $\bar{5}_1^0$ and 5_1^0 gets the weak scale VEV leading to a splitting in the masses of the top quark and the bottom quark. In the case of $\lambda_u = \lambda_d$ as well as small $\tan \beta$ we have to have,

[†]The masses of the weak scale doublets cancel out in the ratio.

$$\gamma_1 \sim \frac{m_b}{m_t} \sim (2.3 - 2.5) \times 10^{-2}, \quad (17)$$

using $m_t = 176$ GeV and $m_b = 4.1 - 4.5$ GeV. Now we are in a position to analyze the parameter space of Eqn. (13) using Eqn. (17) as that of a constraint. Expressing Eqn. (13) in terms of γ_1 we obtain,

$$\Delta\alpha_s^{-1} = \frac{3}{5\pi} \ln \left| \frac{M_3^0}{b_3} \frac{cd}{a_3 e} \gamma_1^{-1} \right|. \quad (18)$$

It is not difficult to lower the prediction of $\alpha_s(m_Z)$ satisfying Eqn. (17); as an example,

$$\frac{a_3}{d} \sim 0.63 \quad ; \quad \frac{a_2}{d} \sim 6.31 \quad ; \quad \frac{e}{c} \sim 6.31 \quad ; \quad \frac{M_3^0}{b_3} \sim 10, \quad (19)$$

we can achieve $\Delta\alpha_s^{-1} = 0.88$ and $\gamma_1 = .025$. This lowers the prediction of $\alpha_s(m_Z)$ to 0.113 from 0.126. This is the lowest value we could achieve keeping all the mass ratios less or equal to 10 and $\gamma_1 = \frac{m_b}{m_t}$. We note that this is not an unique prediction, and $\alpha_s(m_Z)$ could be larger or smaller as well, simply because the threshold corrections depend on the heavy masses of the model. All we are pointing out is that due to the reverse doublet-triplet splitting there exists a range of parameter space which can accomodate the lower values of $\alpha_s(m_Z)$ unlike the minimal SU(5) case.

V. OBSERVATIONS

Before we conclude we observe the following points.

(1) In the minimal case, to achieve a prediction of $\alpha_s = 0.126$ one needs a SUSY spectrum[‡] of the order of 1 TeV or more. However, in the present case, due to the influence of the extra heavy threshold effects, the SUSY spectrum can be as low as the weak scale, and still the prediction of α_s can be kept under desirable control. This makes the present case interesting for the future collider searches.

[‡]This can be understood from the form of the term T_L in Eqn (1). For rigorous details see Ref. [3–5].

(2) In a SO(10) scenario as discussed by Babu and Barr [15] the Higgs scalars in the adjoint representations 45 and 54 of SO(10) lead to the threshold corrections to $\alpha_s(m_Z)$. On the contrary, splitting in the adjoint 24 of SU(5) has no threshold correction to the low energy prediction of $\alpha_s(m_Z)$ [§]. Thus, in a SU(5) theory, the simplest way to alter the prediction of $\alpha_s(m_Z)$ is via the splitting in the extra $5 + \bar{5}$ scalars. Interestingly, the scenario presented above, needs only singlets, fundamentals and adjoint of SU(5), and hence, one is not forced to introduce scalars in the higher dimensional tensorial representations of SU(5) [5,9] to lower the prediction of $\alpha_s^{-1}(m_Z)$.

(3) We digress for a while to the question of R-parity violation [16]. In the minimal SU(5) model the matter-parity violating coupling $\bar{5}_F \bar{5}_F 10_F$ is allowed in the superpotential at the renormalizable level. This coupling leads to the unsuppressed baryon and lepton number violations, which in turn give rise to catastrophic proton decay unless the strength of this coupling is assumed to be vanishing by fiat. In the present model, the Z_9 symmetry forbids this dangerous matter parity violating term offering a natural explanation of R-parity conservation at low energy.

VI. CONCLUSION

To conclude, we have briefly reviewed the fact that in the minimal SU(5) GUT the *prediction* of $\alpha_s(m_Z)$ is at least as high as 0.126, which is inconsistent with the *extraction* of the same from the low energy measurements that give a value around 0.112. Following the construction of an explicit SU(5) model, where the Higgsino mediated proton decay is strongly suppressed [15], the heavy threshold corrections to the prediction of $\alpha_s(m_Z)$ have been calculated. Such a model naturally calls for the introduction of extra $5 + \bar{5}$ Higgs scalars. Small mass splittings between the doublets and the triplets residing in the extra $5 + \bar{5}$

[§] The reason lies in the special group theoretical decomposition of the 24 scalar under the low energy group. After decomposition, the individual contributions cancel each other in Eqn (1).

representations lead to GUT scale threshold corrections to lower the prediction of $\alpha_s(m_Z)$ making it consistent with the values being extracted from the low energy measurements. This model has a natural scenario to explain the large ratio of Yukawa couplings $\lambda_t/\lambda_b \sim 40$. This study shows that if the experimentally extracted value of $\alpha_s(m_Z)$ turns out to be lower than 0.126, and at the same time the dimension five proton decay modes are not observed in SuperKamiokande detector, it will favor the extended SU(5) theory described above over the minimal SU(5) theory.

We acknowledge insightful communications with K. S. Babu, R. N. Mohapatra and J. Pérez-Mercader.

REFERENCES

- [1] M. Shifman, Mod. Phys. Lett. **A10** (1995) 605; TPI-MINN-95-32-T, hep-ph/9511469.
- [2] G. Kane, R. Stuart and J. Wells, Phys. Lett. **B354**, (1995), 350.
- [3] P. Langacker and N. Polonsky, Phys. Rev. **D 47** (1993) 4028; Phys. Rev **D52**, (1995), 3081.
- [4] M. Bastero-Gil and J. Pérez-Mercader, Nucl Phys. **B450**, (1995), 21; Phys. Lett. **B 322** (1994) 355.
- [5] J. Bagger, K. Matchev and D. Pierce, Phys. Lett. **B 348** (1995) 443; P. H. Chankowski, Z. Pluciennik and S. Pokorski, Nucl. Phys. **B 439** (1995) 23.
- [6] L. J. Hall and U. Sarid, Phys. Rev. Lett. **70** (1993) 2673.
- [7] R. Arnowitt and P. Nath, Phys. Rev. Lett. **69** (1992) 725.
- [8] J. Hisano, H. Murayama and T. Yanagida, Phys. Rev. Lett. **69** (1992) 1014; Nucl. Phys. **B 402** (1993) 46.
- [9] K. Hagiwara and Y. Yamada, Phys. Rev. Lett. **70** (1993) 709; Y. Yamada, Z. Phys. **C60** (1993) 83.
- [10] L. Roszkowski and M. Shifman, Phys. Rev. **D 53** (1996) 404.
- [11] D. G. Lee and R. N. Mohapatra, Phys.Rev. **D52** (1995) 4125. B. Brahmachari and R. N. Mohapatra, Phys. Lett. **B 357** (1995) 566. An intermediate scale can also be generated by some alternative mechanisms as described in S. P. Martin and P. Ramond, Phys. Rev. **D51** (1995) 6515.
- [12] A. Masiero, D. V. Nanopoulos, K. Tamkavis and T. Yanagida, Phys. Lett. **115B** (1982) 380.
- [13] For a review, see H.P. Nilles, Phys. Rep. **C110** (1984) 1.

- [14] R. Barbieri and L. J. Hall, Phys. Rev. Lett. **68** (1992) 742.
- [15] K. S. Babu and S. M. Barr, Phys. Rev. **D 48** (1993) 5354; *ibid*, **D 51** (1995) 2463.
- [16] For a review, see L.J. Hall, Mod. Phys. Lett. **A5** (1990) 467.

TABLES

$\overline{5}_1$	5_1	$\overline{5}_2$	5_2	$\overline{5}_3$	5_3	Σ	η_1	η_2	η_3	$\overline{5}_F$	10_F
x^3	1	1	x^6	x^7	x^4	1	x^5	x^7	x^4	x^6	1

TABLE I. The Z_9 charges assigned to various superfields. In our notation $x^9 = 1$